PLASTIC FLOW LOCALIZATION IN SIMPLE SHEAR

V. M. El'kin

UDC 539.38.381

It is well-known that under conditions of low heat loss the heat liberation accompanying the process of plastic deformation can lead to localization of the plastic deformation with formation of so-called adiabatic shear bands. The mechanism for formation of such bands proposed in [1] reduces to failure of the heat liberated in plastic deformation to propagate throughout the volume of the body. The local drop in strength in the heated zone leads to intensification of plastic shear therein, and thus, to still greater heat liberation. The process thus develops catastrophically to the point of material fusion in the localized shear zones. A necessary condition for realization of this model is the presence of significant thermal disorder in the material. The competing processes of deformation hardening and thermal disordering predetermine the existence of a maximum on the deformation curve at some critical deformation value.

Formation of adiabatic shear bands also accompanies processes such as machining, drilling of holes, explosive compaction, etc. A review of experimental observations of plastic flow localization can be found in [2].

Theoretical studies of adiabatic shear have developed in the direction of linear analysis of stability of the solutions of the system of differential equations describing plastic flow in simple shear [3-5], and direct numerical modeling [6, 7] of plastic flow localization.

The present study will consider the effect of the parameters of the defining equation on development of adiabatic shear bands.

1. Formulation of the Problem. We will consider simple shear in an infinite layer of elastoplastic material of thickness d. The lower boundary of the layer is fixed, while the upper boundary moves with a constant velocity v in the direction of the y' axis:

$$y' = Y' + u(x',\ t),\ x' = X',\ z' = Z',\ T' = T'(x',\ t'),$$

where x', y', z' are current and X', Y', Z', initial coordinates of a point; u' is the displacement in the direction of the y' axis; T' is temperature; t', time. The axes y', z' lie in the plane of the layer's lower boundary, while the x' axis is directed normal to that plane.

We write the equations of motion and thermal conductivity in the form

$$\rho\partial^2 u'/\partial t'^2 = \partial\sigma'/\partial x', \quad \rho c\partial T'/\partial t' = \lambda\partial^2 T'/\partial x'^2 + \gamma\sigma'\partial\varepsilon/\partial t' \tag{1.1}$$

with the following boundary and initial conditions:

$$\begin{gathered}\partial u'/\partial t'(0,\ t') = 0,\ \partial u'/\partial t'(d,\ t') = V,\\ u'(0,\ t') = 0,\ u'(d,\ t') = Vt',\ \partial T'/\partial x'(0,\ t') = 0,\ \partial T'/\partial x'(d,\ t') = 0,\\ u'(x',0) = 0, \quad T'(x',0) = T'_0(x'), \quad \partial u'/\partial t'(x',0) = x'V/d.\end{gathered} \tag{1.2}$$

Here ρ, c, λ are the density, specific heat, and thermal conductivity coefficient of the layer material; σ' is the shear stress, and ε is the plastic deformation; γ is the fraction of plastic deformation work transformed into heat; T'_0 is the initial temperature.

We assume that the layer material is elastoplastic, with defining equations of the form

$$\varepsilon_\Sigma = \varepsilon + \sigma'/G, \quad \sigma' = \sigma_0\dot{\varepsilon}'^m\varepsilon^n\exp(-\beta T'), \tag{1.3}$$

where $\varepsilon_\Sigma = \partial u'/\partial x'$ is the total deformation; $\dot{\varepsilon}' = \frac{\partial\varepsilon}{\partial t'} = \frac{\partial\varepsilon_\Sigma}{\partial t'} - \frac{1}{G}\frac{\partial\sigma'}{\partial t'}$ is the plastic deformation rate; σ_0 is a constant, characterizing the strength of the material; m, n are indices of the velocity and deformation sensitivity of the shear stress; β is a coefficient defining temperature disorder; G is the shear modulus.

Chelyabinsk. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 5, pp. 146-151, September-October, 1992. Original article submitted August 15, 1991.

We then transform to the dimensionless quantities:

$$x = x'/d,\ t = t'\dot{\varepsilon}_0,\ u = u'/d,$$
$$\sigma = \sigma'/G,\ T = \beta T',\ \dot{\varepsilon} = \dot{\varepsilon}'/\dot{\varepsilon}_0$$

($\dot{\varepsilon}_0$ = V/d is the mean total deformation rate). We note that in the given case dimensionless time t represents the average of the total deformation across the layer t = $\langle \varepsilon_\Sigma \rangle$.

The system of equations (1.1) with boundary and initial conditions (1.2) then takes on the form

$$\partial^2 u/\partial t^2 = q\partial\sigma/\partial x,\ \partial T/\partial t = r\partial^2 T/\partial x^2 + s\sigma\dot{\varepsilon};$$
$$\partial u/\partial t\,(0,\ t) = 0,\ \partial u/\partial t\,(1,\ t) = 1,\ u(0,\ t) = 0,\ u(1,\ t) = t, \quad (1.4)$$

$$\partial T/\partial x(0,\ t) = 0,\ \partial T/\partial x(1,\ t) = 0,\ u(x,\ 0) = 0,\ T(x,\ 0) = T_0(x),\ \partial u/\partial t(x,\ 0) = x\dot{\varepsilon}_0, \quad (1.5)$$

where $r = \lambda/(\rho cVd)$; $\eta = \sigma_0\dot{\varepsilon}_0^m/G$; $s = G\gamma\beta/(\rho c)$; $q = G/(\rho V^2)$.

We write defining equations (1.3) in a form considering the possible process of unloading in some part of the layer:

$$\sigma = \varepsilon_\Sigma - \varepsilon,$$
$$\dot{\varepsilon} = (\sigma/\eta)^{1/m}\ \varepsilon^{-n/m}\exp(T/m),\quad \text{if}\quad \dot{\varepsilon}_\Sigma > 0, \quad (1.6)$$
$$\dot{\varepsilon} = 0,\quad \text{if}\quad \dot{\varepsilon}_\Sigma \leqslant 0.$$

Inhomogeneous plastic flow of the layer material and its further localization are initiated by an initial inhomogeneous temperature profile with amplitude δ: $T_0(x) = \delta \sin^8(\pi x)$.

2. Calculation Procedures. For numerical solution of Eqs. (1.4)-(1.6) we constructed a nonuniform spatial grid containing N intervals with symmetrical compression toward the layer center x_+:

$$\Delta x_i = x_{i+1} - x_i\ (i = 1,\ 2,\ \ldots,\ N),$$
$$x_1 = 0,\ x_{N+1} = 1,\ x_{N/2+1} = x_+ = 0{,}5.$$

The decrease in grid step size with approach to the center is specified in the lower half of the layer by a rule $\Delta x_{i+1} = a\Delta x_i$ where i = 1, 2, ..., N/2 − 1. The grid in the upper half of the layer is obtained by mirror reflection.

The value of a is chosen such that in the plastic flow localization region there are not less than N_* grid points. To characterize the thickness of the localization layer we use the quantity $H_{\dot{\varepsilon}}$, defined from the equation $\dot{\varepsilon}_+ H_{\dot{\varepsilon}} = \langle\dot{\varepsilon}\rangle$ where ($\dot{\varepsilon}_+$, $\langle\dot{\varepsilon}\rangle$ are the maximum (at the center of the layer) and the mean over the layer of the plastic deformation rate). Method 10*, established by trial and error that the optimum value for N_* is N_* = 10 (for N = 100).

On the time scale two grids were used, one inserted in the other. The step of the first nonuniform grid was chosen from the condition of best description by the solutions obtained in the region where they change abruptly. The step of the second (inserted into the first) uniform grid was determined from the requirement of achieving a specified accuracy over the interval of the first grid.

In order to avoid divergence in calculating the plastic deformation rate at t = 0 a nonzero initial plastic deformation value $\varepsilon(x,\ 0) = 10^{-6}$.

3. Calculation Results. In performing the calculations the following parameter values were taken for solid steel: $\rho = 8\cdot10^3$ kg/m³, $c = 5\cdot10^2$ J/(kg·K), λ = 48 J/(K·sec·m), $\beta = 3\cdot10^{-3}$ 1/K, G = 80 GPa, $\sigma_0\varepsilon_0^m$ = 800 MPa, n = 0.05, m = 0.025. The layer thickness $d = 2.5\cdot10^{-3}$ m, with upper boundary motion rate V = 1.25 m/sec, which corresponds to a mean deformation rate $\dot{\varepsilon}_0 = 5\cdot10^2$ 1/sec. Moreover, it was assumed that all the work of plastic deformation is transformed to heat, i.e., γ = 1. The dimensionless parameters in Eqs. (1.4), (1.6) then take on the values: η = 0.01, s = 60, $r = 3.83\cdot10^{-3}$, $q = 6.4\cdot10^9$. The amplitude of the initial temperature inhomogeneity δ = 0.01. This system of initial problem parameters will be referred to below as the base.

The evolution of plastic deformation rate, plastic deformation, and temperature for the base variant of the initial parameters is shown in Fig. 1a-c. A unique feature of the dependence of plastic deformation rate at the layer center on mean total deformation (or time t)

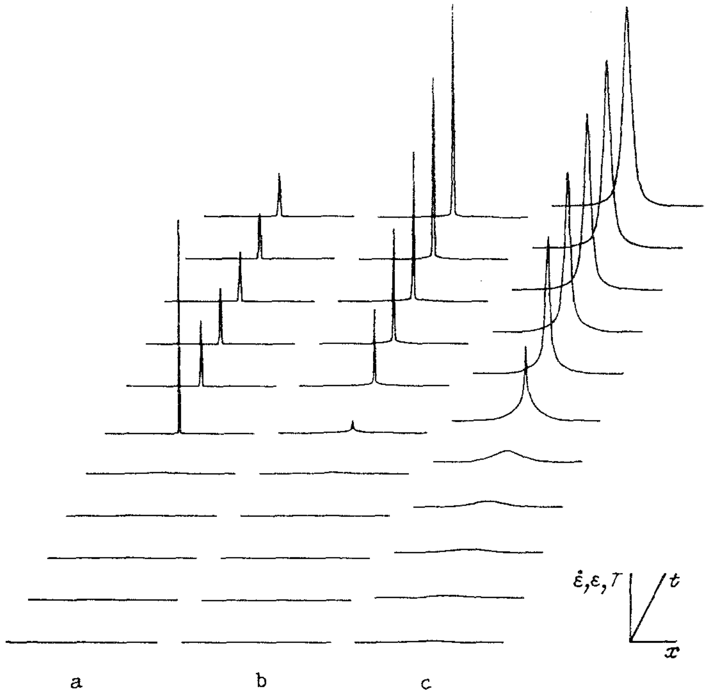

Fig. 1

is the presence of a sharp maximum. Note that the authors of [6, 7], who also modeled the development of an adiabatic shear band, observed no such maximum. In [6] the plastic deformation rate at the layer center $\dot{\varepsilon}_+(t)$ was a monotonic function of time, becoming practically constant at higher t. However in [6] it was assumed in the calculations that the layer material was strictly plastic, and deformation hardening was not considered. Calculation by the present authors with those same assumptions confirmed the results of [6]. Thus, the appearance of a maximum in the $\dot{\varepsilon}_+(t)$ curve is related to use of the more realistic elastoplastic model and consideration of deformation hardening. In [7] defining equations for elastoplastic flow were used, basically similar to those of the present study, but the solution was terminated in the stage of rapid increase in the function $\dot{\varepsilon}_+$.

The numerical calculations produced a solution of the system of differential Eqs. (1.4)-(1.6) for various values of the parameters δ, n, m, s, η, r. In the present note we will limit ourselves to study of the effect of the parameters s, η, r on development of the localization process. Note that over the entire range of s, η, r the shear stress σ(x, t) does not change over layer thickness, i.e., quasistatic loading is realized.

As a characteristic of the degree of localization of some function F(x, t) ($\dot{\varepsilon}$(x, t), ε(x, t) or T(x, t) we introduce the quantity $\xi_F(t) = F_+(t)/F(t)>$, where $F_+(t) = F(0.5, t)$ is the value of the function F(x, t) at the layer center, while $\langle F\rangle(t) = \int_0^1 F(x, t)\,dx$ is the mean value of F(x, t) over the layer thickness.

Analysis of the results reveals some similarity in the effect of the parameters s, η, 1/r on the behavior of the functions $\xi_{\dot{\varepsilon}}(t)$, $\xi_\varepsilon(t)$, $\xi_T(t)$. To present this observation in a clearer manner it is necessary to transform from the dimensionless time t (or the average total deformation over the layer) to the new viable $\tau = \langle\varepsilon\rangle/\langle\varepsilon\rangle_*$ (where ⟨ε⟩ is the mean plastic deformation over the layer, and $\langle\varepsilon\rangle_*$ is the value of that quantity at the point where maximum stress σ_* occurs).

In fact, construction of the functions $\sigma(\tau)/\sigma_*$, $\xi_{\dot{\varepsilon}}(\tau)$, $\xi_\varepsilon(\tau)$, $\xi_T(\tau)$ for various values of s, η, r shows that to a high degree of accuracy (~1%) those functions coincide, if during variation of s, η, r the parameter $\varkappa = s\eta/r$ is maintained.

In Fig. 2 lines 1, 2 show stress normalized to its maximum value $\sigma(\tau)/\sigma_*$, 3, 4 are the degree of localization of plastic deformation rate $\xi_{\dot{\varepsilon}}(\tau)$, and 5, 6 are the degree of localiza-

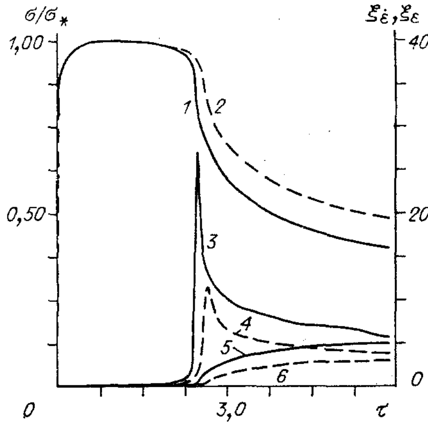

Fig. 2

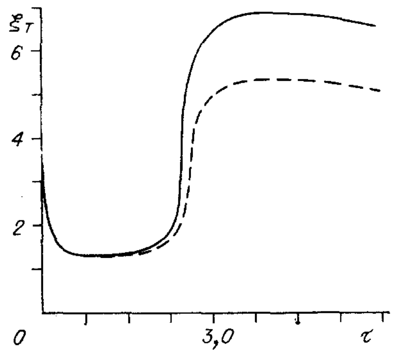

Fig. 3

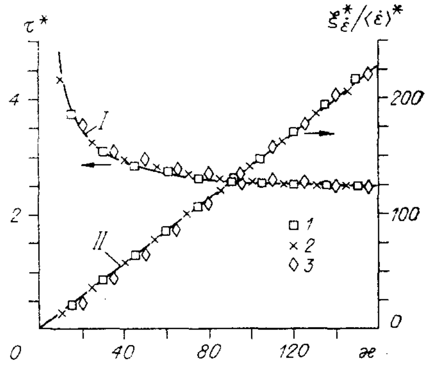

Fig. 4

tion of plastic deformation $\xi_\varepsilon(\tau)$ for $\varkappa$ = 156.6 (curves 1, 3, 5) and 78.3 (curves (2, 4, 6). It is characteristic that severe localization of the plastic deformation rate is accompanied by an abrupt drop in flow stress, as was also noted in [6, 7]. The degree of localization of plastic deformation rate in the peak region significantly exceeds the degree of localization of plastic deformation. However for further deformation these two characteristics approach each other asymptotically.

Temperature undergoes localization to a lesser degree (Fig. 3, solid line being $\varkappa$ = 156.6; dashes, $\varkappa$ = 78.3). The degree of temperature localization $\xi_T(\tau)$ first decreases and then gradually increases after the maximum in stress occurs. The region of abrupt growth in temperature localization also coincides with the region of abrupt drop in flow stress, although the maximum in the degree of temperature localization is significantly smoother than the maximum in the degree of localization of plastic flow rate.

Comparison of the curves $\xi_{\dot\varepsilon}(\tau)$, $\xi_\varepsilon(\tau)$, $\xi_T(\tau)$ with various values of the parameter $\varkappa$ shows that with growth in $\varkappa$ the degree of localization of plastic flow increases, and moreover, at large values of $\varkappa$ the abrupt increase in localization sets in earlier. Thus, $\varkappa$ characterizes the inclination of the material to localization for a given deformation rate. After substitution of expressions for s, η, r the parameter $\varkappa$ can be expressed in the form

$$\varkappa = \gamma\sigma_0\dot\varepsilon_0^m\beta d^2\dot\varepsilon_0/\lambda,$$

so that materials with high strength values $\sigma_0\dot\varepsilon_0^m$, thermal disorder β, and low ability to spread heat over the layer thickness over the characteristic deformation time $\lambda/(d^2\dot\varepsilon_0)$ are more inclined to localization.

For practical purposes, we may take as simple characteristics of the localization process the maximum degree of localization of the plastic deformation rate $\xi_{\dot\varepsilon}^*$ and the value of τ^* at which this maximum is achieved.

Figure 4 shows the dependence of τ^* on $\varkappa$. Variation of $\varkappa$ was carried out by variation of each of the parameters s, η, r while maintaining the base values of the other parameters (points 1-3 correspond to variation of s, η, r). Processing of the data obtained shows that over the entire range $\varkappa$ = 10-156.6 the behavior of τ^* is described by the dependence (line I)

$$\tau^* = 2{,}32(1 + 10/\varkappa). \qquad (3.1)$$

Figure 4 also shows the function $\xi^*_{\dot\varepsilon}/\langle\dot\varepsilon\rangle^*(\varkappa)$, constructed in a similar manner, which over the entire range of $\varkappa$ can be expressed by the equation (line II)

$$\xi^*_{\dot\varepsilon}/\langle\dot\varepsilon\rangle^* = 1{,}42\varkappa \quad (\langle\dot\varepsilon\rangle^* = \langle\dot\varepsilon(\tau^*)\rangle). \qquad (3.2)$$

Equations (3.1), (3.2) can be used to formulate simple localization criteria. If in solving a concrete technological problem the maximum permissible degree of localization $\xi^0_{\dot\varepsilon}$, can be specified, then the inequality $1.42\langle\dot\varepsilon\rangle^*\varkappa \geqslant \xi^0_{\dot\varepsilon}$ can be used as the localization criterion, or, in as much as $\langle\dot\varepsilon\rangle^* \geqslant 1$ $(\partial\sigma/\partial t < 0)$,

$$\varkappa > \xi^0_{\dot\varepsilon}. \qquad (3.3)$$

However, this inequality can be satisfied only if the duration of the loading is sufficiently long that the maximum of the function $\xi_{\dot\varepsilon}(\tau)$ is attained, i.e., inequality (3.3) must be supplemented by the inequality $\tau \geq \tau^*$, or with consideration of Eq. (3.2),

$$\langle\varepsilon\rangle \geqslant 2{,}32\,(1 + 10/\varkappa)\,\langle\varepsilon\rangle_*. \qquad (3.4)$$

One can evaluate the quantity $\langle\varepsilon\rangle_*$ with sufficient accuracy from the homogeneous solutions of Eqs. (1.4)-(1.6) without consideration of the elastic components, since in the region where the maximum of σ_* is attained flow localization is still insignificant, while the overall deformation rate is determined basically by the plastic deformation rate ($\partial\sigma/\partial t \approx 0$). Then

$$\langle\varepsilon\rangle_* \approx \left[\frac{n(1+n)}{s\eta}\right]^{1/(n+1)}, \quad \sigma_* \approx \frac{\eta}{1+n}\left[\frac{n(1+n)}{s\eta}\right]^{n/(n+1)}.$$

Note that criterion (3.3), (3.4) is not absolute, however it can be used for comparison of materials as to their inclination to plastic flow localization.

Thus, numerical modeling of the development of plastic flow perturbation in simple shear in an elastoplastic medium has allowed establishment of certain principles of plastic flow localization, in particular, the presence of a maximum in the dependence of the degree of plastic flow localization on mean deformation. It has been found that strength, thermal disorder, mean deformation rate, and thermal conductivity affect the localization process through a single parameter $\varkappa$. The studies carried out permit formulation of simple criteria for plastic flow localization.

LITERATURE CITED

1. C. Zener and J. H. Hollomon, "Effect of strain rate upon plastic flow of steel," J. Appl. Phys., No. 15 (1944).
2. H. C. Rogers, "Adiabatic plastic deformation," Ann. Rev. Mater. Sci., No. 9 (1979).
3. J. Pan, "Perturbation analysis of shear strain localization in rate sensitive materials," Int. J. Solids Struct., 19, No. 2, (1983).
4. T. J. Burns, "Approximate linear stability analysis of a model of adiabatic shear band formatation," Q. Appl. Math., 43, No. 1 (1985).
5. C. Fressengeas and A. Molinary, "Instability and localization of plastic flow in shear at high strain rates," J. Mech. Phys. Solids, 35, No. 2 (1987).
6. T. W. Wright and J. W. Walter, "On stress collapse in adiabatic shear bands," J. Mech. Phys. Solids. 35, No. 6 (1987).
7. R. C. Batra, "Effect of material parameters on the initiation and growth of adiabatic shear bands," Int. J. Solids Struct., 23, No. 10 (1987).